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CRITERIA FOR THE BREAK-UP OF THIN LIQUID LAYERS FLOWING ISOTHERMALLY OVER SOLID SURFACES

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Abstract—The conditions under which a thin liquid film will tend to completely wet a solid surface over which it is flowing are studied theoretically. Two (very similar) criteria are suggested; one is based on a force balance at the upstream stagnation point of a dry-patch, the other on the minimum total energy rate in a transversely unrestrained stream.

The criteria have been applied to vertical gravity flow of laminar films and to both laminar and turbulent films motivated only by shear forces at the free liquid surface. These latter examples are of special interest with regard to the burn-out in two-phase gas–liquid flows.

Comparisons with experimental evidence appear promising but more detailed experimentation is clearly needed.

NOMENCLATURE

- D, hydraulic diameter of a channel;
- E, rate at which energy crosses a control surface;
- f, friction factor;
- G, function of δ^+, equation (34);
- m, steam quality (fraction of vapour by weight);
- M, rate of mass flow;
- p, static pressure;
- Δp, static pressure difference;
- Re_G, Reynolds number based on gas or vapour alone conditions in a two-phase gas/liquid flow;
- T, force per unit length;
- W, liquid velocity in the direction of flow;
- W_{AG}, average gas or vapour velocity in a channel;
- W_*, friction velocity in turbulent flow;
- x, y, z, rectangular co-ordinates, see Fig. 1;
- X, width of a liquid stream.

Greek symbols

- δ, liquid film thickness;
- θ, angle of contact between liquid and solid;
- μ, liquid viscosity;
- ν, liquid kinematic viscosity;
- ρ, liquid density;
- σ, liquid to air surface tension;
- τ, shear stress.

Suffixes

- c, critical—i.e. at point of film break up;
- G, gas alone;
- L, liquid;
- W, connected with velocity or momentum;
- AG, average, gas phase;
- TP, two-phase;
- δ, at the outer edge of the liquid film;
- σ, connected with surface tension.

1. INTRODUCTION

When a thin liquid film flows over a solid surface under the action of, for example, gravity or a shear applied from a high speed gas stream, dry patches can form and spread. The mechanism initiating a dry patch is not discussed; this paper is concerned only with whether, once a dry patch is formed, it will remain or be re-wetted. The work has arisen in connection with two-phase gas/liquid flow and represents a first stage towards estimating the conditions under which "burn-out" occurs in spray evaporation [1]. The work may also be of value to chemical engineers concerned with designing distillation equipment.

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The criteria presented* lead to theoretical estimates of minimum film thicknesses and flow rates of liquids in motion. Two approaches are made, one based on force balance considerations, the other on a minimum energy (or more strictly a minimum power) analysis. These are shown in section 2; so far they have not been tried out on actual burn-out data. They have, however, been applied to the simpler cases of the vertical flow under gravity of an isothermal laminar film (section 3), the flow under high surface shear in a laminar film (section 4) and to the flow in a turbulent film under high surface shear (section 5). A few comparisons with experimental information appear to be promising.

2. STABILITY CRITERIA

2.1 *Criterion from force-balance at the upstream point of a dry patch*

Consider a film of liquid flowing uniformly over the surface of a flat plate, for example the flow due to gravity down an inclined plane. If the flow rate is reduced sufficiently the stream will break away from the edges of the plate or else disrupt over the central area giving rise to one or more dry patches. An idealized case is depicted in Fig. 1(a) where a (transversely) uniform stream with mean velocity $\bar{W}$ flows onto the upper edge AB of a rectangular plate $ABCD$, and a dry patch $FGHJ$ is formed centrally. The specific flow rate (mass flow rate per unit width) of the liquid leaving the plate across DF and HC is clearly greater than that of the liquid entering across AB, and the precise shape of the dry patch may be connected with the minimum stable film over DF and HC. Consider the forces acting near the point G, the upstream stagnation point of the liquid stream. Figure 1(b) shows a cross-section through the central stream surface EG, and it is assumed in 1(b) that the stream maintains a uniform thickness up to G_s where the meniscus begins. Near the point G, the curvature of the edge of the stream in plan view will in general be very small in comparison with the curvature of G_sG_p in the flow direction and so the surface tension forces arising from the x–z plane curvature will be ignored compared with those arising from the curvature in the y–z plane.

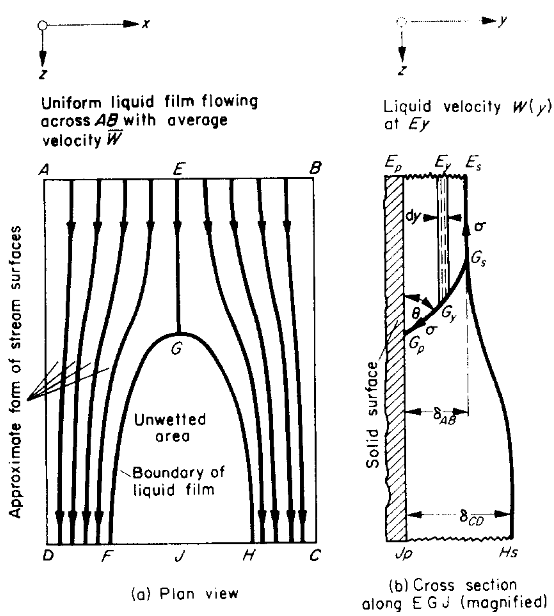

FIG. 1. Dry patch formation in liquid layer flowing over a solid surface.

If the dry patch is stable, the surface tension forces along G_sG_p must balance the fluid pressure over G_sG_p. Under the assumptions implied in Fig. 1, the fluid pressure on the inner surface of G_sG_p exceeds that on the outer surface owing to the conversion of fluid kinetic energy into static pressure. Considering EG as a stream tube of transverse width $\mathrm{d}X$, in an element E_y G_y the velocity $W(y)$ at E_y is gradually reduced to zero at G_y resulting in an excess static pressure at G of

$$\Delta p\,(y) = \rho/2\,[W(y)]^2 \tag{1}$$

The force T_W along G_pG_s due to this resolved in the z-direction will be T_W where

$$T_W = dX \int_0^{\delta_{AB}} \rho[W(y)]^2/2\,\mathrm{d}y \tag{2}$$

The restraining force due to surface tension is

$$T_\sigma = \mathrm{d}X\,.\,\sigma\,.\,(1 - \cos\theta) \tag{3}$$

where σ is the surface tension and θ the contact angle [see Fig. 1(b)].

* These ideas were originally put forward by the authors in Nuclear Research Memorandum Q5, Queen Mary College, London, September 1961.

Thus the point G will be in neutral equilibrium if

$$\sigma(1 - \cos\theta) = \int_0^{\delta_c} \rho/2\,[W(y)]^2\,\mathrm{d}y \qquad (4)$$

It is possible that θ and hence the critical film thickness and minimum wetting rate will depend on whether the experiment is performed by gradually increasing a flow from the completely dry condition or gradually decreasing the flow from a fully wetted condition.

Equation (4) is interesting in its simplicity and its strong dependence on contact angle, the left-hand side of the equation being capable of values ranging from zero to 2σ.

2.2 *Criterion from minimum power in a laterally unrestrained liquid film*

In Fig. 2 liquid is admitted through a slot EF onto the top edge AB of a rectangular plate $ABCD$. The liquid flow under relatively low injection speeds will be roughly as depicted in Fig. 2(a), that is an initial contraction of the stream giving a minimum width at MG followed by an expansion (to LH) and finally a stable width X over the region $LHJK$.

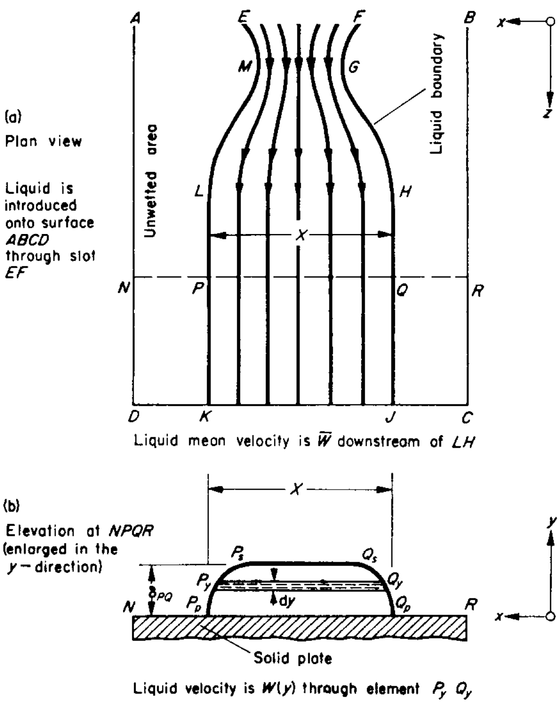

FIG. 2. Flow of a laterally unrestrained liquid layer on a plane surface.

We suggest that the film will attain a stable width X such that the sum of the kinetic energy flow across a plane such as NR plus the surface energy flow will be a minimum. Qualitatively it is clear that a very wide and shallow layer would have a large surface power and a low kinetic power whilst a high kinetic power with a low surface power would result from a deep and narrow stream.

Thus taking a cross-section of the stable stream such as is shown in Fig. 2(b) the rate of kinetic energy flow E_W is

$$E_W = X \int^{\delta_{PQ}} \rho/2\,[W(y)]^3\,\mathrm{d}y \qquad (5)$$

with $W(y)$ the velocity across the element P_yQ_y. If the surface velocity along P_yQ_y is $W(\delta_{PQ})$ then the rate of surface energy flowing is E_σ where

$$E_\sigma = X \,.\, \sigma \,.\, W(\delta_{PQ}) \qquad (6)$$

Thus the criterion leading to the stable stream conditions is simply

$$E_W + E_\sigma \equiv X \int^{\delta_{PQ}} \rho/2\,[W(y)]^3\,\mathrm{d}y + X \,.\, \sigma \,.\, W(\delta_{PQ}) = \text{minimum} \qquad (7)$$

3. LAMINAR FILM FLOWING VERTICALLY UNDER GRAVITY

In a uniform laminar film moving steadily under gravity over a vertical flat surface, the velocity w at a distance y from the wall is given by

$$W = -\frac{\rho g}{2\mu}(y^2 - 2y\delta) \qquad (8)$$

The flow rate per unit width of plate M/X is given by

$$M/X = g\rho^2\,\delta^3/3\mu \qquad (9)$$

3.1 *Minimum thickness from force criterion*

Using equations (4) and (8) we find for the critical thickness:

$$\delta_c = 1{\cdot}72\,[\sigma(1 - \cos\theta)/\rho]^{1/5} \,.\, [\mu/\rho g]^{2/5} \qquad (10)$$

giving a minimum wetting rate of

$$M/X_c = 1{\cdot}69\,(\mu\rho/g)^{1/5}\,[\sigma(1 - \cos\theta)]^{3/5} \qquad (11)$$

3.2 *Minimum thickness from power criterion*

We are interested in the case where the flow is just sufficient to cover the surface of width X, hence if a given flow rate is taken for a given width of plate, equation (7) may be expressed in terms of one constant and one variable—say the film thickness, as variable.

Employing equations (7), (8) and (9) and differentiating with respect to δ in equation (7) we obtain for the critical thickness:

$$\delta_c = 1{\cdot}34\,(\sigma/\rho)^{1/5}\,(\mu/\rho g)^{2/5} \tag{12}$$

The minimum wetting rate is then

$$(M/X_c) = 0{\cdot}803\,(\mu\rho/g)^{1/5}\,\sigma^{3/5} \tag{13}$$

Thus the critical film thicknesses and flow rates derived from the two criteria are almost identical except that the force criterion includes an effect of contact angle whilst the power criterion does not.

3.3 *Comparisons with experimental data*

Experimental data are available from three sources [2, 3 and 4], but in none of these works were contact angles measured, it is therefore only possible to compare them with the power criterion.

Dukler and Bergelin [2] describe experiments on water films flowing down a vertical plate of polished stainless steel. They were not especially interested in minimum wetting rates, but it is reasonable to assume that their smallest recorded film thicknesses were in fact the smallest that they could produce. Water was used at 77°F and Table 1 shows a comparison between the smallest measured values and those estimated from equations (12) and (13).

Table 1. *Minimum wetting rate and film thicknesses for water flowing over vertical stainless steel plate*

Quantity	Measured [2]	Estimated
Minimum wetting rate (lb/ft h)	261	248
Minimum film thickness (in)	0·012	0·0121

In reference 3, Bressler shows flow rates and film thicknesses for water at 100°C on a steel plate. Table 2 compares the minimum rates with those calculated from the power criterion.

Table 2. *Minimum wetting rate and film thicknesses for water flowing over a vertical steel plate*

Quantity	Measured [3]	Estimated
Minimum wetting rate (lb/ft h)	103	166
Minimum film thickness (in)	0·0063	0·0075

Norman and McIntyre [4] measured minimum wetting rates on the inside of a 1 in diameter smooth copper pipe with various temperatures both of the water and of the tube metal surface. The isothermal results [4, Fig. 4] are very different from those estimated by the present formulae, as shown in Table 3, although the minimum wetting rates measured with the wall at 100°C are very close to the estimates for isothermal conditions.

Table 3. *Minimum wetting rates for water flowing inside a* 1 *in diameter smooth copper pipe*

Water mean temperature (°C)	Surface temperature (°C)	M.W.R. measured [4] (lb/ft h)	M.W.R. estimated
30	30	25	244
30	100	250	
45	45	25	223
45	100	225	
60	60	25	206
60	110	158	
75	75	25	192
75	100	125	

Thus of the three sets of data, one agrees well with the power criterion (Table 1), the second gives an estimate of M.W.R. which is 60 per cent too big, and in the third series the estimates from minimum power are about ten times too great. It is not possible from these comparisons to evaluate the applicability of the power criterion in general. One would expect there to be considerable contact angle changes for a subcooled liquid in contact with a surface whose temperature is increased up to the saturation temperature of the liquid.

The force criterion, equation (11), has been used to deduce the contact angles which would give agreement with the data already discussed. The results of this analysis are shown in Table 4.

Table 4. Contact angles to give agreement between test data and force criterion

Test	Contact angle
[2, Table 1]	45°
[3, Table 2]	30°
[4, Table 3]	20° (at 30°C)

No special significance is attached to these estimates, but they do show how powerful an influence the angle of contact has: a change in contact angle from 45° to 20° decreases the M.W.R. by a factor of about ten. It should be noted that for a completely non-wetting liquid/solid system (contact angle 180°) the force criterion predicts a minimum wetting rate 3·2 times greater than the power criterion.

4. LAMINAR FILM MOTIVATED BY SURFACE SHEAR ONLY

If a laminar liquid film is flowing under the influence of surface shear so great that the weight of the liquid is not significant, the velocity in the film is given by

$$W = \tau y/\mu \tag{14}$$

and at the surface of the liquid the velocity is

$$W(\delta) = \tau\delta/\mu \tag{15}$$

The total specific flow rate is given by

$$M/X = \int_0^\delta \rho W \, \mathrm{d}y = \rho\tau\delta^2/2\mu \tag{16}$$

4.1 *Minimum thickness from force criterion*

Applying equation (4) to the profile of equation (14) leads directly to the relation:

$$\delta_c = 1{\cdot}82\,[\sigma(1-\cos\theta)/\rho]^{1/3}\,(\mu/\tau)^{2/3} \tag{17}$$

and the minimum wetting rate is

$$[M/X]_c = 3{\cdot}30\,(\rho\mu/\tau)^{1/3}\,[\sigma(1-\cos\theta)]^{2/3} \tag{18}$$

The equations (17) and (18) exhibit similar forms to equations (10) and (11).

4.2 *Minimum thickness from power criterion*

Using equations (7), (14) and (15) and differentiating with respect to δ as before, we obtain:

$$\delta_c = 1{\cdot}59\,(\sigma/\rho)^{1/3}\,(\mu/\tau)^{2/3} \tag{19}$$

and the minimum wetting rate is

$$[M/X]_c = 2{\cdot}52\,(\rho\mu/\tau)^{1/3}\,\sigma^{2/3} \tag{20}$$

4.3 *Comparisons with experimental data*

In two-phase flows (gas–liquid systems) in pipes, various experimenters [5, 6] have measured liquid film thicknesses and pressure drops and, although they have not investigated minimum wetting rates, it is interesting to see how the predicted minima compare with their measurements. The application of the formulae of sections 4.1 and 4.2 is not possible without making several assumptions, since in a two-phase flow there is an unknown quantity of liquid in the vapour regime. Moreover, in applying the power criterion, the analysis requires modification since the two-phase surface shear is dependent on the film thickness. Before making estimates for two-phase annular flows in channels, the power criterion will be reconsidered taking into account the dependence of surface shear on film thickness and employing a relationship recently proposed [7] by D. C. Roberts (see also Appendix of the present paper).

4.4 *Minimum wetting conditions from power criterion, with surface shear dependent on film thickness*

Consider the two-phase annular flow in a vertical channel, and in which most of the liquid is flowing in a film on the wall(s) and the vapour (or gas) occupies the central part of the channel. According to Roberts [7] the friction factor may be written as

$$f_{TP} = f_G + 1{\cdot}42\,[\delta/D - 5/Re_G \,.\, (2/f_G)^{1/2}] \quad (21)$$

and the shear stress at the surface of the liquid layer is

$$\tau_i = f_{TP} \,.\, 1/2 \,.\, \rho_G W_{AG}^2 \quad (22)$$

In these equations the suffix G refers to the 'gas alone' conditions. More details of the pressure drop correlation are given in the Appendix.

τ is not independent of the liquid film thickness δ so that the minimum condition is

$$\mathrm{d}E/\mathrm{d}\delta \equiv (\partial E/\partial\tau)_\delta\,(\partial\tau/\partial\delta)_{W_{AG}} + (\partial E/\partial\delta)_\tau = 0 \quad (23)$$

where we are considering a fixed gas velocity but variable liquid rate.

From equations (21), (22) and (23) we obtain

$$1{\cdot}42\delta_c^4/Df_{TP} + \delta_c^3 - 4\sigma\mu^2/\rho\tau^2 = 0 \quad (24)$$

The equation may be solved by a trial and error procedure, remembering that both τ and f_{TP} are dependent on δ_c. Having evaluated δ_c, equation (16) is still valid for obtaining the minimum wetting rate.

4.5 *Estimates of minimum wetting rates for an air/water system*

Consider a two-phase, air and water, flow inside a vertical tube of 1·25 in bore. Let the M.W.R. values be estimated for gas velocities corresponding to Reynolds numbers of 20 000, 60 000 and 200 000. The temperature is taken to be 70°F and the mean air pressure 1·0 psig.

Calculations according to equation (24) lead to results as shown in Table 5.

Table 5. Minimum wetting rates for annular air/water flow in $1\frac{1}{4}$ in bore pipe. T–70°F; p–1·57 psia

Re_G	2×10^4	6×10^4	2×10^5
W_{AG} (ft/s)	29	86	286
M_G (lb/h)	72·5	217·5	725
$1/2\,\rho_G W_g^2$ (lbl/ft²)	32·7	294	3270
δ_c (in)	0·0195	0·0075	0·0023
f_{TPc}	—	0·0112	0·0064
$\rho g\delta/\tau$	4·4	0·39	0·018
M_{Lc} (lb/h)	—	74	42
$M_G/(M_{Lc} + M_G)$	—	0·747	0·943
Re_{Lc}	—	380	210

At the lowest of the three Reynolds numbers the calculation is not valid because the weight of the liquid per unit area of film surface is 4·4 times greater than the surface shear.

Reference 6 describes measurements in a $1\frac{1}{4}$ in pipe with annular air/water flow under the same approximate conditions as the example in Table 5. The tests were not, however, concerned with minimum wetting rates, but film thickness measurements were made. These indicate that the values calculated are too high; for example, with a gas flow of 597 lb/h (corresponding to a gas Reynolds number of $1{\cdot}7 \times 10^5$) liquid films were measured with flow rates down to 15 lb/h so that the estimate in Table 5 at $Re = 2 \times 10^5$ is at least three times too big. Similarly at a gas flow at 225 lb/h ($Re = 6{\cdot}4 \times 10^4$) film thicknesses were measured down to 11 lb/h liquid flow, so the estimates at $Re = 6 \times 10^4$ are at least seven times too great.

5. TURBULENT FILM MOTIVATED BY SURFACE SHEAR ONLY

5.1 *Velocity distribution*

Assume that the liquid in the film obeys the von Kármán universal profile, i.e.

$$W^+ = y^+, \qquad 0 < y^+ < 5 \quad (25a)$$

$$W^+ = 5 \log y^+ - 3{\cdot}05, \qquad 5 < y^+ < 30 \quad (25b)$$

and

$$W^+ = 5{\cdot}5 + 2{\cdot}5 \log y^+, \qquad y^+ > 30 \quad (25c)$$

where W^+ and y^+ are defined by

$$W^+ = W/W_*; \qquad y^+ = yW_*/\nu \quad (26)$$

and where

$$W_* = (\tau/\rho)^{1/2} \quad (27)$$

The use of these equations in thin liquid layers has been discussed by several authors, in particular by Dukler and Bergelin [2] and by Murgatroyd [8]. Whilst the profiles are known to be imperfect in the thin film application, they have not yet been superseded and have the merit of simplicity.

5.2 *Minimum thickness from force criterion*

The force criterion [equation (4)] can be written in terms of the dimensionless velocity and distance as

$$\sigma (1 - \cos \theta) = \mu W_*/2 \int_0^{\delta_c^+} W^{+2} \, dy^+ \quad (28)$$

Putting the velocity distributions of equations (25) into equation (28) gives the following relationships:

(a) for laminar films ($\delta^+ < 5$)

$$\delta_c^+ = [6\sigma (1 - \cos \theta)/\mu W_*]^{1/3} \quad (29)$$

which is in fact the same equation as obtained for a laminar film in section 4 but with rearranged parameters.

(b) for a film in the transition zone ($5 < \delta_c^+ < 30$)

$$25\, \delta_c^+ \log^2 \delta_c^+ - 80{\cdot}5\, \delta_c^+ \log \delta_c^+ + 89{\cdot}8\, \delta_c^+ - 83{\cdot}3 = 2\, \delta (1 - \cos \theta)/\mu W_* \quad (30)$$

(c) for a fully turbulent film ($\delta_c^+ > 30$)

$$6{\cdot}25\, \delta_c^+ \log^2 \delta_c^+ + 15\, \delta_c^+ \log \delta_c^+ + 15{\cdot}25\, \delta_c^+ - 1084 = 2\sigma (1 - \cos \theta)/\mu W_*. \quad (31)$$

Equations (28) to (31) may be written generally as:

$$I_2 (\delta^+) = 2\, \sigma (1 - \cos \theta)/\mu W_* \quad (32)$$

and the function $I_2 (\delta^+)$ is given in Table 8.

5.3 *Minimum thickness from power criterion*

A. Shear at surface independent of film thickness. Writing equations (5) to (7) in terms of the dimensionless parameters, we have:

$$\frac{M}{\mu \int_0^{\delta^+} W^+ \, dy^+} [1/2 \mu\, W_*^2 \int_0^{\delta^+} W^{+2} \, dy^+ + \sigma W_* W_c^+] = = \text{minimum} \quad (33)$$

With the shear taken as constant, W_* the friction velocity is also a constant so that differentiation with regard to δ is the same as with respect to δ^+. Hence the criterion can finally be put as:

$$G (\delta^+) = 2\, \sigma/W_* \quad (34)$$

where

$$G (\delta^+) \equiv \frac{(W_\delta^+)^3 \int_0^{\delta^+} W^+ \, dy^+ - W_\delta^+ \int_0^{\delta^+} (W^+)^3 \, dy^+}{(W_\delta^+)^2 - \dfrac{d W_\delta^+}{d\delta^+} \int_0^{\delta^+} W^+ \, dy^+} \quad (35)$$

and values are given for G in Table 8 for $\delta^+ = 0$ to $\delta^+ = 100$.

5.4 *Minimum thickness from power criterion*

B. Shear stress dependent on film thickness. Where the shear stress is dependent on the liquid film thickness, as in the case of the two-phase gas/liquid flows, the criterion becomes more complex.

The total energy rate E_T [equation (33)] is a function of δ^+ and W_*, so that differentiating with respect to δ gives

$$\frac{dE_T}{d\delta} = \left(\frac{\partial E_T}{\partial \delta^+}\right)_{W_*} \cdot \frac{d\delta^+}{d\delta} + \left(\frac{\partial E_T}{\partial W_*}\right)_{\delta^+} \frac{dW_*}{d\delta} \quad (36)$$

Taking the Roberts correlation [equation (21)] together with equation (27), and differentiating, leads to

$$\frac{dW_T}{d\delta} = \frac{0{\cdot}71}{Df_{TP}} W^* \quad (37)$$

Differentiating equation (26) and using equation (37) gives

$$\frac{d\delta^+}{d\delta} = \frac{W_*}{\nu}\left[1 + \frac{0{\cdot}71}{f_{TP}} \cdot \frac{\delta}{D}\right]. \quad (38)$$

Employing these results in equations (33) and (36) leads to the desired criterion. The equation obtained is as follows:

$$\left.\begin{aligned} &(W_\delta^+)^3 \int_0^{\delta^+} W^+\,dy^+ - W_\delta^+ \int_0^{\delta^+} (W^+)^3\,dy \\ &+ \frac{2\sigma}{\mu W_*}\left[\frac{dW_\delta^+}{d\delta^+} \int_0^{\delta^+} W^+\,dy^+ - (W_\delta^+)^2\right] \\ &+ \frac{1{\cdot}42\,\nu \int_0^{\delta^+} W^+\,dy^+}{f_{TP}\,W_* D\left(1 + \frac{0{\cdot}71\,\delta}{f_{TP}\,D}\right)} \left[\int_0^{\delta^+} (W^+)^3\,dy^+ \right. \\ &\left. + \frac{\sigma}{\mu W_*} W_\delta^+\right] = 0. \end{aligned}\right\} \quad (39)$$

Table 8 facilitates the solution of equation (39).

5.5 *Estimates of minimum wetting rates for a steam/water system*

Consider now the heat-transfer tests reported by Collier [1]. The test section and its operating conditions are listed below:

Test section:		
size	0·866 in × 0·623 in annulus	
flow area	0·001973 ft^2	
hydraulic diameter	0·0202 ft	
perimeter	0·39 ft	
Fluid condition:		
pressure	15·9 psia	
temperature	216°F	
Physical properties:	Water	Steam
density lb/ft^3	59·7	0·0402
viscosity lb/ft h	0·67	0·0318
surface tension	58 dyn/cm	

Two estimates have been made of minimum liquid flows for various steam velocities using equations (34) and (39) respectively. In these estimates it has been assumed that there is no entrainment.

Equations (34) and (39) both stem from the power criterion. The latter corresponds to the real case of two phase flow where the shear stress is dependent on film thickness whilst the former does not. However, equation (34) is less complex than (39) and is therefore useful to give approximate solutions which can serve as starting points in solving equation (39). Its application to the conditions outlined gives results which are presented in Table 6. It would appear that in annular flow with little liquid entrainment the film minimum wetting rates decrease as the gas flows increase. In experiments where the distribution of liquid between the film and the gas core is not known measurements of minimum wetting rates may appear to contradict this result since, with increasing gas flows the quantity of liquid in the core generally increases.

Taking into account friction factor variations, the power criterion for film stability is:

$$A - \frac{2\sigma}{\mu W_*} B^+ \frac{1{\cdot}42\,\nu\,I_1\left[I_3 + \frac{\sigma}{\mu W_*} W^+\right]}{f_{TP}\,W_*\,D\left[1 + \frac{0{\cdot}71\,\delta}{f_{TP}\,D}\right]} = 0 \quad (40)$$

Table 6. Application of equation (34) *to conditions of reference* 1

W_{AG} (ft/s)	200	400	600
R_{eG}	18,400	36,800	55,200
δ_c^+	50	34	26
δ (in)	0·00355	0·00163	0·00097
δ/D	0·0146	0·0057	0·0040
Re_{Lc}	2300	1350	910
M_G (lb/h)	57	114	171
M_L (lb/h)	150	87	59
M_T (lb/h)	207	201	230
m (quality)	0·28	0·57	0·75

Table 7. Application of equation (39) *to reference* 1

W_{AG} (ft/s)	200	400	600
Re_G	18,400	36,800	55,200
δ_c^+	24	20·5	18·1
δ (in)	0·00218	0·00112	0·00074
δ/D	0·00896	0·00462	0·00304
Re_{Lc}	803	628	562
M_G	57	114	171
M_L	54	41	36·5
M_T	111	155	207·5
m	0·515	0·735	0·825

where

$$A(\delta^+) \equiv (W_\delta^+)^3 \int_\delta^{\delta^+} W^+ \,dy^+ - W_\delta^+ \int_0^{\delta^+} (W^+)^3 \,dy^+ \tag{41a}$$

$$B(\delta^+) \equiv (W_\delta^+)^2 - \frac{dW^+}{d\delta^+} - \frac{dW^+}{d\delta^+} \int_0^{\delta^+} W^+ \,dy^+ \tag{42b}$$

$$I_1(\delta^+) \equiv \int_0^{\delta^+} W^+ \,dy^+ \tag{42c}$$

$$I_3(\delta^+) \equiv \int_0^{\delta^+} (W^+)^3 \,dy^+ \tag{42d}$$

Equations (40) and (41) are the same as equations (39) but are written in a way more convenient for computation. The functions of δ^+ are given in Table 8.

Solution of the equations for the case outlined at the start of this section leads to the results shown in Table 7.

Table 8. Integrations of the von Kármán turbulent velocity profiles

δ^+	W^+	I_1	I_2	I_3	A	B	G
1	1	0·5	0·3333	0·25	0·25	0·5	0·5
2	2	2·0	2·667	4	8	2·0	4·0
3	3	4·5	9·000	20·25	60·75	4·5	13·5
4	4	8·0	21·33	64·00	256	8	32·0
5	5	12·5	41·66	156·2	779·1	12·47	62·47
6	5·91	17·97	71·62	320·7	1811	19·94	90·83
7	6·68	24·27	111·4	572·2	3411	27·28	125·0
8	7·35	31·29	160·7	919·1	5658	34·42	164·4
9	7·94	38·94	219·3	1367	8614	41·35	208·3
10	8·46	47·14	286·6	1920	12330	48·05	256·6
11	8·94	55·85	362·4	2580	16830	54·53	308·7
12	9·37	65·01	446·3	3349	22160	60·80	364·5
13	9·77	74·59	538·0	4228	28330	66·86	423·8
14	10·15	84·55	637·3	5217	35360	72·73	486·2
15	10·49	94·87	743·8	6316	43260	78·42	551·6
16	10·81	105·50	857·3	7526	52030	83·94	619·8
17	11·12	116·5	977·6	8845	61690	89·31	690·7
18	11·40	127·5	1104	10270	72230	94·52	764·2
19	11·67	139·3	1238	11810	83660	99·59	840·1
20	11·93	151·1	1377	13450	95980	104·50	918·3
21	12·17	163·1	1522	15200	109200	109·30	998·6
22	12·41	175·4	1673	17060	123300	114·00	1081
23	12·63	187·9	1830	19020	138200	118·6	1166
24	12·84	200·7	1992	21090	154100	123·1	1252
25	13·04	213·6	2159	23250	170800	127·4	1340
26	13·24	226·8	2332	25530	188400	131·7	1431
27	13·43	240·1	2510	27900	206900	135·9	1522
28	13·61	253·6	2693	30370	226200	140·0	1616
29	13·79	267·3	2881	32940	246400	144·0	1711
30	14·00	281·2	3073	35610	273400	172·7	1584
31	14·08	295·2	3270	38380	284400	174·6	1629
32	14·16	309·4	3470	41200	295600	176·5	1675
33	14·24	323·6	3672	44060	307000	178·3	1722
34	14·32	337·8	3875	46980	318700	180·1	1770
35	14·39	352·2	4081	49930	330700	181·9	1818
36	14·46	366·6	4289	52930	342900	183·6	1867
37	14·53	381·1	4500	55980	355300	185·3	1917
38	14·59	395·7	4712	59060	367900	187·0	1968
39	14·66	410·3	4925	62190	380800	188·6	2019
40	14·72	425·0	5141	65360	363800	190·2	2071
41	14·78	439·7	5359	68580	407100	191·8	2123
42	14·84	454·6	5578	71830	420600	193·3	2176
43	14·90	469·4	5800	75120	434300	194·8	2230
44	14·96	484·4	6023	78450	448300	196·3	2284
45	15·02	499·4	6247	81810	462400	197·8	2338
46	15·07	514·4	6474	85220	476700	199·2	2393
47	15·13	529·5	6702	88660	491200	200·6	2449
48	15·18	544·7	6931	92140	505900	202·0	2505
49	15·18	559·9	7162	95650	520800	203·4	2561
50	15·28	575·1	7395	99200	535900	204·7	2618
51	15·33	590·4	7629	102800	551200	206·1	2675
52	15·38	605·8	7865	106400	566700	207·4	2733
53	15·43	621·2	8102	110100	582300	200·7	2791
54	15·47	636·6	8342	113700	598100	209·9	2849
55	15·52	652·1	8581	117500	614100	211·2	2908
56	15·56	667·7	8823	121200	630300	212·4	2967

Table 8—continued

δ^+	W^+	I_1	I_2	I_3	A	B	G
57	15·61	683·2	9065	125000	646600	213·6	3027
58	15·65	698·9	9310	128800	663100	214·8	3087
59	15·69	714·5	9555	132700	679800	216·0	3147
60	15·74	730·3	9802	136600	696600	217·2	3207
61	15·78	746·0	10050	140500	713600	218·3	3268
62	15·82	761·8	10300	144400	730700	219·5	3329
63	15·86	777·7	10550	148400	748000	220·6	3391
64	15·90	793·5	10800	152400	765500	221·7	3453
65	15·94	809·4	11060	156400	783100	222·8	3515
66	15·97	825·4	11310	160500	800900	223·9	3577
67	16·01	841·4	11570	164600	818900	225·0	3640
68	16·05	857·4	11820	168700	836900	226·0	3703
69	16·09	873·5	12080	172800	855200	227·1	3766
70	16·12	889·6	12340	177000	873500	228·1	3829
71	16·16	905·7	12600	181200	891200	229·1	3893
72	16·19	921·9	12860	185400	910700	230·2	3957
73	16·23	938·1	13130	189700	929500	231·2	4021
74	16·26	954·4	13390	194000	948500	232·2	4086
75	16·29	970·6	13650	198300	967600	233·1	4150
76	16·33	986·9	13920	202600	986800	234·1	4215
77	16·36	1003·0	14190	207000	1006000	235·1	4280
78	16·39	1020·0	14460	211400	1026000	236·0	4346
79	16·42	1036·0	14730	215800	1045000	236·9	4412
80	16·46	1065·0	15000	220300	1065000	237·9	4477
81	16·49	1069	15270	224700	1085000	238·8	4543
82	16·52	1085	15540	229200	1105000	239·7	4610
83	16·55	1102	15810	233700	1125000	240·6	4676
84	16·58	1119	16090	238300	1145000	241·5	4743
85	16·61	1135	16360	242900	1166000	242·4	4810
86	16·64	1152	16640	247400	1186000	243·3	4877
87	16·66	1168	16920	252100	1207000	244·1	4944
88	16·69	1185	17190	256700	1228000	245·0	5012
89	16·72	1202	17470	261400	1249000	245·9	5079
90	16·75	1219	17750	266100	1270000	246·7	5147
91	16·78	1235	18030	270800	1291000	247·5	5215
92	16·80	1252	18320	275500	1312000	248·4	5284
93	16·83	1269	18600	280300	1334000	249·2	5352
94	16·86	1286	18880	285000	1355000	250·0	5421
95	16·88	1303	19170	289800	1377000	250·8	5489
96	16·91	1320	19450	294700	1399000	251·6	5550
97	16·94	1336	19740	299500	1420000	252·4	5627
98	16·96	1353	20030	304400	1442000	253·2	5697
99	16·99	1370	20310	309300	1464000	254·0	5766
100	17·01	1387	25480	314200	1487000	254·8	5836

The critical liquid flow rates are less than those from equation (43) but the trend with increasing steam flow rate is the same.

6. DISCUSSION

Considerations of the forces acting at a break-point in a liquid film lead to theoretical values of the minimum wetting rates which are similar in form to those obtained from minimum power considerations in liquid streams not restrained laterally.

For water flowing down a vertical surface under gravity, the power criterion calculations agree well with measurements of Dukler and Bergelin (Table 1) and moderately well with those of Bressler (Table 2). There appear to be large discrepancies between the estimates and the measurements of Norman and McIntyre. To the authors' knowledge the latter experiments were made first producing complete wetting conditions and then reducing the flow rate of liquid. It is possible that with the calm, laboratory conditions the films were in a metastable state and that the same low values might not have been achieved if the flow had been increased from zero. Equally likely is the possibility that the contact angle was a low one in those tests and in that case it would not be expected that the power criterion would give a true estimate. In fact, in all the comparisons made and in all the estimates calculated in this paper it has only been possible to use the power criterion since no values of solid/liquid contact angles have been reported in the test work done until now.

The examples (sections 4 and 5) of liquid films under high surface shear are included in view of their relevance to two-phase gas/liquid flows. In this field there are so far no experiments at all connected directly with minimum wetting rates, although measurements have been made [6] of film thicknesses in a 1·25 in pipe. The calculations, based on laminar films (Table 5) lead to minimum film thicknesses in the right order of magnitude, but the minimum wetting rates estimated are 3 to 7 times too big.

The use of the von Kármán profiles for turbulent liquid films lead to somewhat complex formulae and equations; these can be solved without too much difficulty, making use of the various functions tabulated (Table 8). Application of the formulae to a small annulus with steam/water flows gives estimated minimum thicknesses (Table 7) in the range 0·7 to 2 thousandths of an inch. So far as can be inferred from analysis of the test results, these thicknesses are of the correct order of magnitude. Little else can be said about the comparisons with tests until such time as data is obtained which includes measurements of contact angles.

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APPENDIX

The friction factor for two-phase annular flows

It is shown in reference 7 that the pressure losses in two-phase annular flows can be best analysed in terms of an interfacial friction factor f_{TP}, calculated on the "gas alone" properties, i.e.

$$\Delta p = f_{tp} \cdot 1/2\, \rho_G W_{AG}^2 \cdot \frac{4Z}{D} \pm \rho_G g Z \quad \text{(A1)}$$

Where W_{AG} is the average gas velocity ignoring the presence of liquid, and ρ_G is the gas density. The second term in (A1) is the hydrostatic head due to the gas core and is taken positively or negative for downward and upward flows respectively.

For liquid films having a mean calculated thickness δ which is less than the thickness of the laminar sublayer of the gas alone stream, the two-phase pressure drop is no different from that of the gas by itself, i.e. if

$$\delta/D < 5/Re_G\,(2/f_G)^{1/2}, \quad f_{TP} = f_g \quad \text{(A2)}$$

For annular flow regime, provided that less than about 20 per cent of the moisture entrained in the gas core,

$$f_{TP} = f_g + 1{\cdot}42 \left[\frac{\delta}{D} - \frac{5}{Re_G}\left(\frac{2}{f_G}\right)^{1/2}\right]. \quad \text{(A3)}$$

The estimated liquid film thicknesses may be readily estimated from the liquid Reynolds number and a knowledge of the velocity profile. In reference 3, von Kármán profiles were assumed and appeared to be adequate.

Whilst the friction factor equation (A3) is a simple one, its use is limited at present owing to the difficulty of determining the limits of the annular regime.

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Résumé—Les conditions, sous lesquelles un film liquide mince tendra à mouiller complètement une surface solide sur laquelle il coule, sont étudieées théoriquement. Deux critères (très semblables) sont suggérés; l'un est basé sur un bilan de forces au point d'arrêt amont d'une partie sèche, l'autre sur le débit minimal d'énergie totale dans un écoulement non borné latéralement.

Les critères ont été appliqués à l'écoulement vertical par gravité de films laminaires et à la fois à des films laminaires et turbulents mus seulement par des forces de cisaillement à la surface libre du liquide. Ces derniers exemples sont d'un intérêt spécial eu égard à la caléfaction dans des écoulements diphasiques du type gaz-liquide.

Des comparaisons avec l'éxpérience apparaissent pleines de promesses mais on a besoin évidemment d'une expérimentation plus détaillée.

Zusammenfassung—Die Bedingungen, unter welchen ein dünner Flüssigkeitsfilm, der über eine feste Oberfläche fliesst, diese vollständig benetzt, werden theoretisch untersucht. Zwei (sehr ähnliche) Kriterien werden vorgeschlagen; das eine beruht auf dem Kräftegleichgewicht am oberen Staupunkt eines Trockenbereiches, das andere auf der minimalen Gesamtenergieänderung in einem seitlich nicht geführten Strom.

Die Kriterien wurden für senkrechte Strömungen unter dem Einfluss der Schwerkraft angewandt, sowohl auf laminare Filme als auch auf laminare und turbulente Bewegungen, die allein von Zähigkeitskräften an der freien Flüssigkeitsoberfläche herrühren. Letztere Beispiele sind besonders für den burn-out in Gas-Flüssigkeitsströmen interessant.

Vergleiche mit experimentellen Erscheinungen sind vielversprechend, jedoch sind weitere, ins einzelne gehende Versuche erforderlich.

Аннотация—Теоретически изучены условия, при которых тонкая пленка жидкости стремиться полностью смочить поверхность твердого тела, которое она обтекает. Предложены два критерия. Один критерий основан на равновесии сил в критической точке сухого участка вверх по потоку, другой—на минимальном расходе энергии в поперечном сечении для несжимаемого потока.

Критерии применяли для ламинарных пленок, в поле силы тяжести на вертикальной поверхности, а также для ламинареых и турбулентных пленок, вызванных только силами сдвига на свободной поверхности жидкости. Особый интерес представляют последние случаи для критических нагрузок в двухфазных потоках газа и жидкости.

Сравнение с экспериментальными данными многообещающее, но требует проведения более детального исследования.